UNIT -2
Chapter: II

ASSIGNMENT PROBLEM

Introduction:

Assignment Problem is a special type of linear programming problem where the objective is to minimise the cost or time of completing a number of jobs by a number of persons. The assignment problem in the general form can be stated as follows:

"Given n facilities, n jobs and the effectiveness of each facility for each job, the problem is to assign each facility to one and only one job in such a way that the measure of effectiveness is optimised (Maximised or Minimised)."

Several problems of management have a structure identical with the assignment problem. For example:

Example I:

A manager has four persons (i.e. facilities) available for four separate jobs (i.e. jobs) and the cost of assigning (i.e. effectiveness) each job to each person is given. His objective is to assign each person to one and only one job in such a way that the total cost of assignment is minimised.

Example II:

A manager has four operators for four separate jobs and the time of completion of each job by each operator is given. His objective is to assign each operator to one and only one job in such a way that the total time of completion is minimised.

Example III:

A tourist car operator has four cars in each of the four cities and four customers in four different cities. The distance between different cities is given. His objective is to assign each car to one and only one customer in such a way that the total distance covered is minimized.

Hungarian Method:

Although an assignment problem can be formulated as a linear programming problem, it is solved by a special method known as Hungarian Method because of its special structure. If the time of completion or the costs corresponding to every assignment is written down in a matrix form, it is referred to as a Cost matrix. The Hungarian Method is based on the principle that if a constant is added to every element of a row and/or a column of cost matrix, the optimum solution of the resulting assignment problem is the same as the original problem and vice versa. The original cost matrix can be reduced to another cost matrix by adding constants to the elements of rows and columns where the total cost or the total completion time of an assignment is zero. Since the optimum solution remains unchanged after this reduction, this assignment is also the optimum solution of the original problem. If the objective is to maximise the effectiveness through Assignment, Hungarian Method can be applied to a revised cost matrix obtained from the original matrix.

Balanced Assignment Problem:

Balanced Assignment Problem is an assignment problem where the number of facilities is equal to the number of jobs.

Unbalanced Assignment Problem:

Unbalanced Assignment problem is an assignment problem where the number of facilities is not equal to the number of jobs. To make unbalanced assignment problem, a balanced one, a dummy facility(s) or a dummy job(s) (as the case may be) is introduced with zero cost or time.

Dummy Job/Facility:

A dummy job or facility is an imaginary job/facility with zero cost or time introduced to make an unbalanced assignment problem balanced.

Infeasible Assignment:

An Infeasible Assignment occurs in the cell (i, j) of the assignment cost matrix if ith person is unable to perform jth job. It is sometimes possible that a particular person is incapable of doing certain work or a specific job cannot be performed on a particular machine. The solution of the assignment problem should take into account these restrictions so that the infeasible assignments can be avoided. This can be achieved by assigning a very high cost to the cells where assignments are prohibited.

Steps involved in solving minimisation problems:

Step 1:

See whether number of rows are equal to number of columns. If yes, problem is balanced one; if not, then add a Dummy Row or Column to make the problem a balanced one by allotting zero value to each cell of the Dummy Row or Column, as the case may be.

Step 2:

Row Subtraction: Subtract the minimum element of each row from all elements of that row.

Note: If there is zero in each row, there is no need for row subtraction.

Step 3:

Column Subtraction: Subtract the minimum element of each column from all elements of that column.

Note: If there is zero in each column, there is no need for column subtraction.

Step 4:

Draw minimum number of horizontal and/or vertical lines to cover all zeros. To draw minimum number of lines the following procedure may be followed:

1. Select a row containing exactly one uncovered zero and draw a vertical line through the column containing this zero and repeat the process till no such row is left.

2. Select a column containing exactly one uncovered zero and draw a horizontal line through the row containing the zero and repeat the process till no such column is left.

Step 5:

If the total lines covering all zeros are equal to the size of the matrix of the Table, we have got the optimal solution; if not, subtract the minimum uncovered element from all uncovered elements and add this element to all elements at the intersection point of the lines covering zeros.

Step 6:

Repeat Steps 4 and 5 till minimum number of lines covering all zeros is equal to the size of the matrix of the Table.

Step 7:

Assignment: Select a row containing exactly one unmarked zero and surround it by, ‘and draw a vertical line through the column containing this zero. Repeat this process till no such row is left; then select a column containing exactly one unmarked zero and surround it by, and draw a horizontal line through the row containing this zero and repeat this process till no such column is left.

Note: If there is more than one unmarked zero in any row or column, it indicates that an alternative solution exists. In this case, select anyone arbitrarily and pass two lines horizontally and vertically.

Step 8:

Add up the value attributable to the allocation, which shall be the minimum value.

Step 9:

Alternate Solution: If there are more than one unmarked zero in any row or column, select the other one (i.e., other than the one selected in Step 7) and pass two lines horizontally and vertically. Add up the value attributable to the allocation, which shall be the minimum value.

Steps problems involved solving in maximisation:

Step 1:

See whether number of rows is equal to number of columns. If yes, problem is a balanced one; if not, then adds a dummy row or column to make the problem a balanced one by allotting zero value to each cell of the dummy row or column, as the case may be.

Step 2:

Derive Profit Matrix by deducting cost from revenue.

Step 3:

Derive Loss Matrix by deducting all elements from the largest element.

Step 4:

Follow the same Steps 2 to 9 as involved in solving Minimisation Problems.